General instructions for Students: Whatever be the notes provided, everything must be copied in the Maths copy and then do the HOMEWORK in the same copy.

CLASS – IX

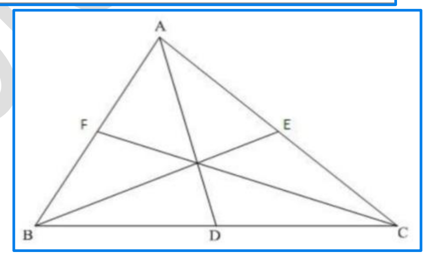
MATHEMATICS

12. Pythagoras Theorem (Part – II) *EXERCISE – 12*

22. If AD, BE and CF are medians of Δ ABC, prove that

$$3\,(AB^2 + BC^2 + CA^2) = 4\,(AD^2 + BE^2 + CF^2)$$

> Since in any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median bisecting it.



Solution : $AB^2 + AC^2 = 2(\frac{1}{2}BC)^2 + 2AD^2$

$\Rightarrow \quad AB^2 + AC^2 = \frac{1}{2}BC^2 + 2AD^2 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

Similarly, $AB^2 + BC^2 = \frac{1}{2}AC^2 + 2BE^2 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Similarly, $BC^2 + AC^2 = \frac{1}{2}AB^2 + 2CF^2 \ldots\ldots\ldots\ldots\ldots\ldots (iii)$

Adding (i), (ii) and (iii), $2\,(AB^2 + BC^2 + AC^2) = \frac{1}{2}\,(BC^2 + AC^2 + AB^2) + 2\,(AD^2 + BE^2 + CF^2)$

$$\Rightarrow \quad 2(AB^2 + BC^2 + AC^2) = \frac{(AB^2 + BC^2 + AC^2) + 4\,(AD^2 + BE^2 + CF^2)}{2}$$

$$\Rightarrow \quad 4\,(AB^2 + BC^2 + AC^2) = (AB^2 + BC^2 + AC^2) + 4\,(AD^2 + BE^2 + CF^2)$$

$$\Rightarrow \quad 4\,(AB^2 + BC^2 + AC^2) - (AB^2 + BC^2 + AC^2) = 4\,(AD^2 + BE^2 + CF^2)$$

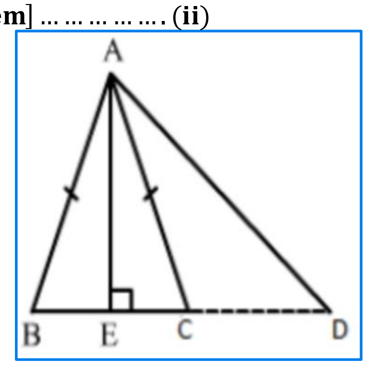
$\Rightarrow \quad 3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$ **Proved.**

26. In an isoseles triangle ABC, AB = AC and D is apoint on BC produced. Prove that

$$AD^2 = AC^2 + BD \times CD.$$

Solution : In Δ AED, $\angle AED = 90^\circ$ $\quad AD^2 = AE^2 + DE^2$ [By Pythagoras Theorem] $\ldots\ldots\ldots\ldots\ldots\ldots$ (i)

In Δ AEC, $\angle AEC = 90^\circ$ $\quad AC^2 = AE^2 + CE^2$ [By Pythagoras Theorem] $\ldots\ldots\ldots\ldots\ldots$ (ii)



Subtracting (ii) from (i), $AD^2 - AC^2 = DE^2 - CE^2$

$$\Rightarrow \quad AD^2 - AC^2 = (CE + CD)^2 - CE^2$$

$$\Rightarrow \quad AD^2 - AC^2 = CE^2 + CD^2 + 2 \times CE \times CD - CE^2$$

$$\Rightarrow \quad AD^2 - AC^2 = CD^2 + 2 \times CE \times CD$$

$$\Rightarrow \quad AD^2 - AC^2 = (CD + 2 \times CE) \times CD$$

$\Rightarrow \quad AD^2 - AC^2 = (CD + BC) \times CD \qquad [\because\ 2\ CE = BC]$

$\Rightarrow \quad AD^2 - AC^2 = BD \times CD$

$\Rightarrow \quad AD^2 = AC^2 + BD \times CD$ **Proved.**

Chapter Test

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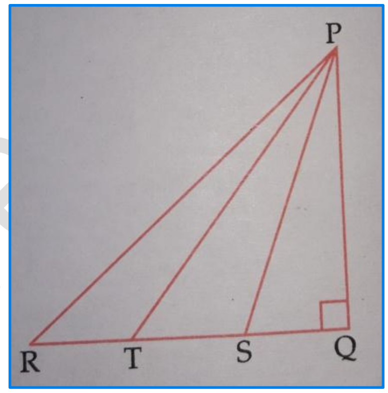
4. In the adjoining figure, Δ PQR is right angled at Q and points S and T trisect side QR.

Prove that $8PT^2 = 3PR^2 + 5PS^2$



Solution : $QS = ST = TR = x$ (say)

In Δ PQT, $\angle Q = 90°$

$PT^2 = PQ^2 + QT^2$ [By Pythagoras Theorem]

$\Rightarrow PT^2 = PQ^2 + (2x)^2 \quad [\ QT = 2x\ ]$

$\Rightarrow PT^2 = PQ^2 + 4x^2$

$\Rightarrow 8\ PT^2 = 8\ PQ^2 + 32x^2 \ldots\ldots\ldots\ldots (i)$ [Multiplying both sides by 8]

In Δ PQR, $\angle Q = 90°$

$PR^2 = PQ^2 + QR^2$ [By Pythagoras Theorem]

$\Rightarrow PR^2 = PQ^2 + (3x)^2 \quad [\ QR = 3x\ ]$

$\Rightarrow PR^2 = PQ^2 + 9x^2$

$\Rightarrow 3\ PR^2 = 3\ PQ^2 + 27x^2 \ldots\ldots\ldots\ldots (ii)$ [Multiplying both sides by 3]

In Δ PQS, $\angle Q = 90°$

$PS^2 = PQ^2 + QS^2$ [By Pythagoras Theorem]

$\Rightarrow PS^2 = PQ^2 + x^2 \quad [\ QS = x\ ]$

$\Rightarrow 5\ PS^2 = 5\ PQ^2 + 5x^2 \ldots\ldots\ldots\ldots (iii)$ [Multiplying both sides by 5]

Adding (ii) and (iii), $3\ PR^2 + 5\ PS^2 = 3PQ^2 + 27x^2 + 5PQ^2 + 5x^2$

$\Rightarrow 3\ PR^2 + 5\ PS^2 = 8\ PQ^2 + 32x^2$

$\Rightarrow 3\ PR^2 + 5\ PS^2 = 8\ PT^2$ [Using (i)] **Proved.**

HOMEWORK

Exercise – 12

Question numbers : 17, 18, 19(b), 20, 21, 23 and 24

Chapter test : $1(b)$ *and* 3